**Homework 7 Solutions**

**(Energy)**

**Problem 1.** An 8cm3 block of ice at temperature T = -10°C sits in microwave. You turn the microwave on high, at 500W, and wait for 30s. Describe what you’ve got when you open the microwave door. Take cice = 2.1 kJ/kg°C, cwater = 4.18 kJ/kg°C, cwater vapor = 2.0 kJ/kg°C; Lf = 333 kJ/kg, and Lv = 2256 kJ/kg. And finally take the density of ice to be ρ = 920 kg/m3.

The mass of our cube is:



And so the change in energy of the cube will be:

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We don’t know what the final temperature will be, or even if it will all melt or vaporize. Supposing it all vaporizes, let’s calculate the final temperature:

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Since we get a negative temperature change as a vapor, that means it didn’t all vaporize. So now let’s recalculate, assuming only some of it, Δm, vaporizes. Then,

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Since we get a sensible answer, we can conclude that the ice has only partially vaporized. So what we’ll have is 0.0042kg of water vapor, and 0.0030kg of water, at 100°C.

**Problem 2.** In order to escape Earth’s gravitational field, an object would have to have a velocity of about 11km/s. This applies to rockets and gas molecules equally. If a gas molecule’s average speed is greater than 10% of the escape speed, then there is a high enough chance it will acquire enough speed, via random collisions, to exceed the escape velocity every once and a while (and once is enough).

(a) What is the average speed of a He molecule at the typical temperature T = 300K. What does the tell you about the likelihood of finding He gas in our atmosphere?

Recall,



So,



Since this is greater than 10% the escape velocity, we may expect that whatever amount of He gas initially present in our atmosphere is long gone.

(b) What is the average speed of a N2 molecule?

Speed of N2 is:



(c) Of an O2 molecule?

Speed of an O2 is:



(d) of an H2O molecule (water vapor)?

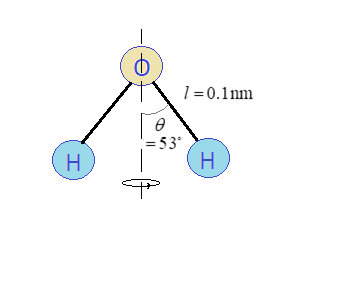
And H2O speed is:



(e) Should we be worried about losing all of our air, and our oceans (to evaporation)?

These are less than 10% the escape velocity, so we’re OK. Yay!

**Problem 3.** A water vapor molecule (H2O) floats in the atmosphere at room temperature T = 25°C.



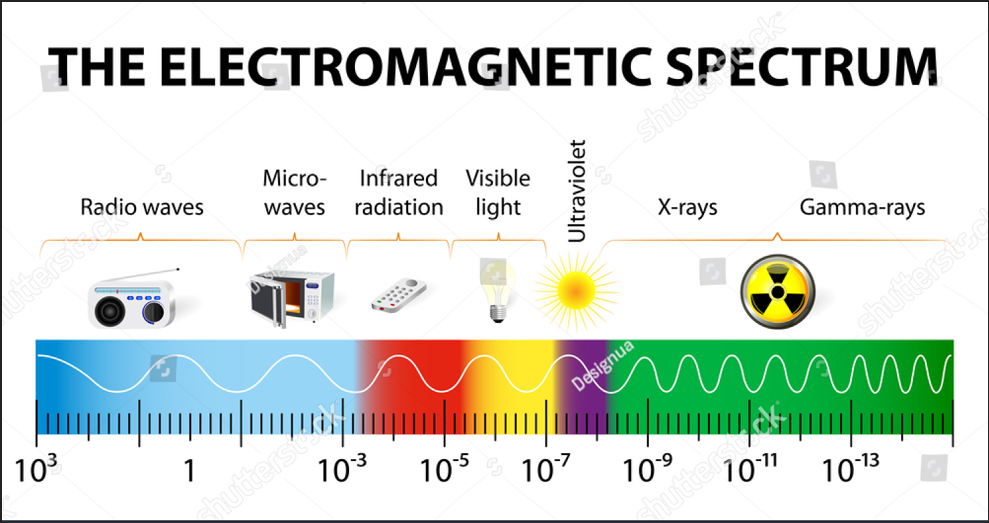
(a) What is its moment of inertia about the axis shown?

Well,



check

(b) Estimate the molecule’s frequency of rotation about this axis (also recalling that f = ω/2π). As we discussed in class, in the context of thermal radiation, when charges oscillate with a given frequency, they emit electromagnetic waves at the same frequency. What part of the EM spectrum does this frequency correspond to? Note the chart’s scale is in terms of wavelength.



From the equipartition theorem, we have:



From which it follows that:



that’s 2.2 trillion times per second! And then,



which corresponds to the infrared spectrum. Everything at terrestial temperatures emits infrared radiation.

check

(d) What is the total energy of our water vapor molecule?

The total energy of the molecule is:



(e) If we fill a 1000cm3 balloon with water vapor to a pressure P = 50 kPa, what will be the gas’s total energy?

Energy of all the molecules is, recalling PV = NkBT:



**Problem 4.** Gonna consider the relationship between our E(T,V,N) formulas for gasses and solids, and their specific heat capacities.

(a)The energy of an ideal gas, at high temperatures, is E(T,V,N) = (f/2)NkBT. Based on this, derive an expression for the specific heat capacity (at constant volume) of an ideal gas, in terms of its molar mass mmolar and the gas constant R = 8.31 J/mol∙K. May recall that NAkB = R.

So,



(b) Evaluate this formula for Oxygen gas. Compare to its typical value cO gas = 0.66 kJ/kg∙K.

Well the molar mass of O2 is about 32g/mol. So,



Agreement is quite good.

(c) An approximate formula for the energy of an ideal solid, in the high temperature limit, is, as was discussed in class: E(T,V,N) = fNkBT/2 + (B/2υ)(V-υ)2, where f = 6, B is the bulk modulus, and υ is the volume at 0 pressure, more or less. Using this E formula, derive an expression for the specific heat capacity (at constant volume) of such a solid, in terms of its molar mass mmolar, and the ideal gas constant R (recall NAkB = R).

Using the same procedure, we have:



(d) Evaluate this formula for (solid) gold (baby). Compare to its typical value cgold = 0.126kJ/kg∙K.

Well, the molar mass of gold is: 197g/mol. So we’d expect:



Perfecto.

(e) So these E(T,V,N) formulas work well at high temperatures. But it turns out they really suck at low temperatures. As we’ll see when we discuss entropy, the heat capacity of *anything* should go to zero when T → 0, rather than simply remain constant for all temperatures as these formulas predict. Resolving this dilema was the initial impetus towards the discovery of quantum mechanics. Turns out that as the temperature of an object goes down, it begins to manifest more and more of its wave-nature. In any event, eventually the more succesful ‘Debye’ model predicted the heat capacity of an object would look like this at low temperatures:



Where TD is the Debye temperature for that object, which is more or less the temperature at which its wave-like nature begins to show up. The Debye temperature of gold is TD = 165K. So how much would the energy of 2kg of gold change as it was heated from 0 to 100K?

This is given by:



**(1st Law of Thermodynamics)**

**Problem 5.** Solar radiation coming from the Sun delivers heat to the Earth’s oceans, and they evaporate (and later precipitate too), cooling themselves off. Estimate the rate of evaporation of water (in kg/s) from the oceans by modeling Earth as a sphere of water, and accounting for just these two heat transfer mechanisms. You can assume that the intensity of sunlight is roughly 1200 W/m2, and that εsolar ≈ 1.

Looks like this,



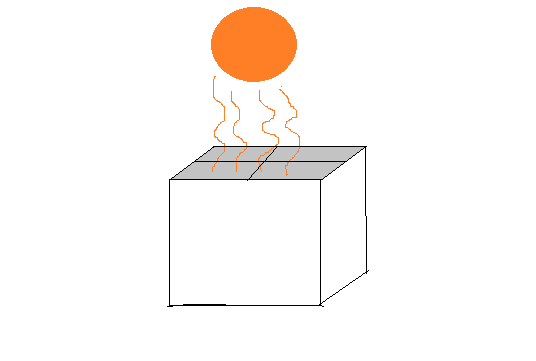
**Problem 6.** Sunlight is incident on asphalt at an angle of 20° with respect to the vertical. The sunlight’s intensity is 1250 W/m2, and 8% of it is reflected from the surface. The surrounding air temperature is about 25°C. What temperature will the asphalt equilibrate to, assuming the ashphalt loses heat mostly through convection, and gains through solar radiation? You may take the convection coefficient to be 15 W/m2°K.

Yeah,



**Problem 7.** Suppose a cubic shed has a window for a roof, and is under direct sunlight. Let the sunlight intensity be I = 1200 W/m2 and 10% be reflected from the window. The walls and floor of the shed are made of wood, with thermal conductivity k = 0.12 W/m∙K and thickness 5cm. The thermal conductivity of the window is about 0.95 W/m∙K, and has thickness 1cm. Finally the ambient air temperature is 20°C. What will be the equilibrium temperature of the shed? You’ll be considering solar radiation and conduction (through all six surfaces of the shed) here.

So,





**Problem 8.** Earth is about 1.5×1011m from the Sun, and the intensity of sunlight here is about 1200W/m2. Neptune’s mean distance from the Sun is about 4.5×1012m. (a) What do you expect sunlight’s intensity to be there? (b) Assuming that Neptune’s temperature is maintained by the balance between solar and thermal radiation, what do you expect it’s average temperature to be? You may assume Neptune radiates heat across its entire surface, and take the ε’s to be 1.

Sunlight’s intensiity would be:



And then,



**(Entropy)**

**Problem 9.** What is the overall change in entropy of the block of ice in problem 1?

So simple,

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**Problem 10.** Suppose you have 0.28kg of N2 gas, in a 0.02m3 container, at temperature 25°C. Then you let it freely expand into another insulated container with twice the volume.

(a) What will be its new temperature?

The same as the old one, since:



(b) What will be its change in entropy?

Even easier,



(c) Consider the more accurate model of a gas, the Van der Waals equation. In such, nitrogen gas is characterized by the constants a = 0.1361 J∙m3/mol2, and b = 3.85×10-5 m3/mol. What is the change in temperature of the gas when it expands into the new container? And explain your result.

So now,



The temperature drops, because the gas loses kinetic energy because it gains potential energy as it expands.

(d) And now the change in entropy?

This is,



**Problem 11.** What is the overall change in entropy of the 2kg of gold in problem 4e, using the Debye model of heat capacity?

Will you just stop assigning problems? Even I get tired of solving them. Nope, I care about my students too much to let their education be substandard. Learn by calculating.



**(1st and 2nd Laws of Thermodynamics)**

**Problem 12.** Consider a ball which falls on the ground. When it does so, it converts most of the kinetic energy, ΔKE, it had into internal energy, ΔEint.. And consequently, its temperature will increase – a fact you can verify with a sensitive enough thermometer. But we do not see the reverse occur, and object resting on the floor suddenly getting colder by converting some internal energy ΔEint. into kinetic energy ΔKE, and thereby jumping off the floor. So consider an object with mass m, and specific heat capacity c, undergoing either of these processes. Using the first and second laws of thermodynamics, show that the former scenario is ok, but the latter is not. You needn’t bother with any heat flow, or work; we’re only considering direct energy conversion.

Well 1st law says:



And 2nd law says:



Now from 2nd law, the only way ΔSint. will be positive (or zero) as required is if Tf > Ti → ΔT > 0 → ΔKE is negative. So we can only lose KE and convert to internal energy, not gain KE at the cost of internal energy.

**Problem 13.** When we place any two objects in thermal contact, the first law of thermodynamics doesn’t *require* that they equilibrate to the same temperature: it would still be satisfied if the hot one got hotter, and the cold one colder, as long as energy were conserved. It is the second law of thermodynamics that requires the temperatures equalize. We did a quick formal demonstration of this fact in class, but let’s make a more tangible illustration. For simplicity, consider two objects of equal mass m, and specific heat capacities, c, but different initial temperatures T1i and T2i. Let these be insulated (no Q), isolated (no W) and in thermal contact. And let’s allow that they settle into new temperatures T1f and T2f, which we know *should* be equal, but which we won’t assume. Prove that, subject to the constraint provided by the 1st law of thermodynamics, entropy will be maximized if and only if T1f = T2f.

So first law requires:



And second law requires:



One way to maximize a function subject to a constraint it to plug the constraint into the function, and then differentiate w/r to the remaining variable and set to zero. So solving for, say, T2f, in terms of T1f, using the 1st law constraint we get: T1f = (T1i + T2i) – T2f. Plugging into the entropy and maximizing we get:



And now plugging back into our T1f equation we get:



And so we see that the two final temperatures are equal. If you’re familiar with the technique of Lagrange multipliers, this can be done more quickly. Calling S the left hand side of the final entropy equation, and E the constraint, we would evaluate:



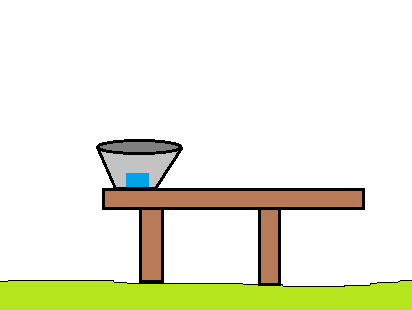
And so we see that T1f and T2f are the same. And if we cared to evaluate what the temperatures actually were, we would plug them into the E constraint:



And so we’d conclude:



**Problem 14.** A 50g ice cube (cice = 2kJ/kg∙C) at -20°C is placed in a well insulated (with foam or something) 1.5kg aluminum bucket (cAl = 0.92 kJ/kg∙C) whose initial temperature is 30°C. But you accidentally knock the bucket off the table, and it falls 1.2m to the floor. And then your dog gets excited by the commotion and knocks the bucket around a few times, doing 100J of work on it, and in the process displacing the bucket lid a little, which allows 750J of heat to escape to the 30°C outside air before you put the lid back on.



(a) What will be the final equilibrium temperature of the ice + bucket?

Even easiest,



(b) What will be the overall change in entropy for this situation? You may assume the air doesn’t appreciably change its temperature when absorbing the 750J. Is the 2nd law of thermodynamics satisfied?

And change in entropy is (putting all the energies in kJ):



So 2nd law is satisfied.

**Problem 15.**  You and your friend through snowballs at each other – very gluey snowballs designed specifically for this problem. You throw your m1 = 1kg snowball with a speed v1 = 28m/s; your friend throws her m2 = 2kg snowball with a speed v2 = 16m/s in the opposite direction. They collide and stick together.

(a) Assuming their initial temperature is T = 0C, and it’s a T = 0C day, how much of the snow melts?

First we must use conservation of momentum,



And then we use the 1st law:



(b) What is their change in entropy? Is the second law satisfied?

Change in entropy is:



As it is positive, the 2nd law is satisfied.